First Semester MCA Degree Examination, June/July 2016
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions.*

1 a. For any statements p, q prove the following:
   i) $\sim(p \downarrow q) \Leftrightarrow (\sim p \uparrow \sim q)$ ii) $\sim(p \uparrow q) \Leftrightarrow (\sim p \downarrow \sim q)$ (04 Marks)

   b. Define Tautology. Verify the compound propostition $\{(p \vee q) \wedge [(p \to r) \wedge (q \to r)]\} \to r$ is a tautology or not.   (05 Marks)

   c. Define dual of logical statement. Verify the principle of duality for the following logical equivalence, $[\sim(p \wedge q) \to \sim p \vee (\sim p \vee q)] \Leftrightarrow (\sim p \vee q)$.   (06 Marks)

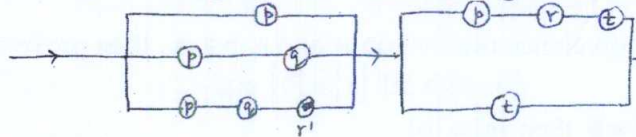   d. Simplify the following switching network without using truth tables: (05 Marks)

Fig. Q1 (d)

2 a. Verify the validity of the following argument : "If Rochelle gets the supervisor's position and works hard, then she will get a raise. If she gets a raise, then she will buy a car. She has not purchased a car. Therefore, either Rochelle did not get the supervisor's position or she did not work hard."   (06 Marks)

   b. Define open sentence. Write the negation of "All integers are rational numbers and some rational numbers are integers".   (02 Marks)

   c. Define converse, inverse and contrapositive of an implication. Hence find converse, inverse and contrapositive for "$\forall x, (x > 3) \to (x^2 > 9)$" where universal set is the set of real numbers R.   (06 Marks)

   d. Give : (i) a direct proof, (ii) an indirect proof and (iii) proof by contradiction, for the following statement: "If n is an odd integer then n + 9 is an even integer".   (06 Marks)

3 a. Find the sets A and B if $A \cap B = \{2,4,7\}$, $A \cup B = \{1, 2, 3, 4, 5, 7, 8, 9, 10\}$ and $A - B = \{1, 8\}$ (02 Marks)

   b. Among the integers 1 to 200, find the number of integers that are, (i) divisible by 2 or 5 or 9, (ii) not divisible by 5, (iii) not divisible by 2 or 5 or 9, (iv) divisible by 5 and not by 2 and 9.   (08 Marks)

   c. Using laws of set theory simplify the following : (i) $A \cap (B - A)$ (ii) $\overline{(A \cup B \cap C)} \cup \overline{B}$ (05 Marks)

   d. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:
   i) There is no restriction in the choice.
   ii) Two particular persons will not attend separately.
   iii) Two particular persons will not attend together.   (05 Marks)

4 a. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers, prove that $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$.   (05 Marks)

   b. Give a recursive definition for each of the following integer sequence:
   i) $a_n = 7n$ ii) $a_n = 3n + 7$ iii) $a_n = 2 - (-1)^n$ (05 Marks)

   c. Define GCD of two positive integers. Find the GCD of 231 and 1820 and express it as a linear combination.   (05 Marks)

   d. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day.   (05 Marks)

5   a.   Define Cartesian product of two sets. For any three non-empty sets A, B and C. Prove that
         $A \times (B - C) = (A \times B) - (A \times C)$ **(05 Marks)**

    b.   Define stirling's number of second kind. If $|A| = 7$, $|B| = 4$, find the number of onto
         functions from A to B. Hence find S(7, 4).   **(05 Marks)**

    c.   Prove that a function $f : A \rightarrow B$ is invertible if and only if it is one-one and onto.   **(05 Marks)**

    d.   State the pigeonhole principle. Let ABC be an equilateral triangle with AB = 1 cm. show
         that if we select five points in the interior of the triangle, there must be at least two points
         whose distance is less than $\frac{1}{2}$ cm.   **(05 Marks)**

6   a.   Let A = {1, 2, 3, 4} and R be a relation on A defined by xRy iff x divides y or y divides x.
         Write down R as a relation of set of ordered pairs, relation matrix M_R and digraph of R. Also
         find in-degree and out-degree of each vertex.   **(05 Marks)**

    b.   Define partition of a set. If R is a relation defined on A = {1, 2, 3, 4} by R = {(1, 1), (1, 2),
         (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4, 4)}, determine the partition induced by the equivalence
         relation.   **(05 Marks)**

    c.   Let R be an equivalence relation on A and a, $b \in A$, then prove the following:
         i) $a \in [a]$ ii) aRb iff $[a] = [b]$ and
         iii) if $[a] \cap [b] \neq \phi$ then $[a] = [b]$.   **(05 Marks)**

    d.   Let A = {a, b, c}, B = P(A) where P(A) is the power set of A. Let R be a subset relation on
         A. Prove that (B, R) is a POSET and draw its Hasse diagram. Is it a lattice?   **(05 Marks)**

7   a.   Define the following with an example:
         i) Regular graph   ii) Complete graph   iii) Bipartite graph.   **(06 Marks)**

    b.   Define Isomorphism of two graphs. Verify the following graphs are isomorphic or not.
         (05 Marks)

Fig. Q7 (b)

    c.   Explain Konigsberg bridge problem.   **(05 Marks)**

    d.   Define Hamilton graph. Verify the following graph is Hamilton graph or not.   **(04 Marks)**
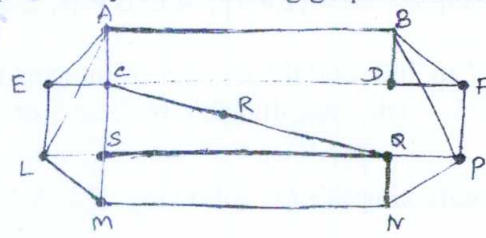

Fig. Q7 (d)

8   a.   Define chromatic number. Find the chromatic number of the graph shown below:
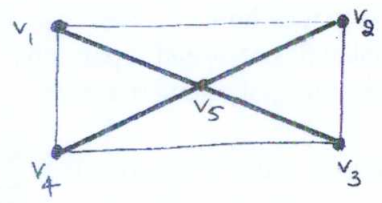   **(05 Marks)**

Fig. Q8 (a)

    b.   Define a Tree. Show that a tree with 'n' vertices has n − 1 edges.   **(05 Marks)**

    c.   Define planar graph. Show that the bipartite graphs $K_{2,2}$ and $K_{2,3}$ are planar graphs.
         (05 Marks)

    d.   A class room contains 25 microcomputers that must be connected to a wall socket that has 4
         outlets. Connections are made by using extension cords that have 4 outlets each. Find the
         least number of cords needed to get this computer set up for the class.   **(05 Marks)**

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